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by William B. Fussell and John B. Schutt

Goddard Space Flight Center
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SUMMARY

A definition of "closeness" of fit of a gray body curve to the sun's extraterrestrial spectrum is presented. F. S. Johnson's data are used for the solar spectrum. For the wavelength interval 0.3 to 2.4 microns (pertinent to thermal design), the magnitude of C_1 calculated for use in Planck's black body formula is 8.6504×10^{-17} watt-cm^2 at a temperature of 5742 ± 0.5 °K.

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INTRODUCTION

The problem of simulating the extraterrestrial solar illumination for testing the thermal design of satellites has recently become of wide interest. If the materials for the satellite surface have been selected, the problem reduces to one of matching the absorbed energy from the simulating source—for each material composing the surface—to the absorbed solar energy for that material. However, many materials have spectral absorptivity curves which vary slowly with wavelength between wavelength limits including most of the sun's energy. A solar simulator which matches the sun's spectrum fairly well on a finely resolved basis, and which matches the total energy exactly, will automatically match the solar absorptivities and the simulated absorptivities closely for these materials. To this end, the black body function was used in constructing a gray body curve for which a solar fit was obtained by parametrically varying the temperature.

METHOD

In establishing a closeness of fit criterion for approximating the sun's extraterrestrial spectrum by that of a black body, it seems pertinent at this point to recall the geometric interpretation of the quantities in the black body equation which can be treated as parameters. Consequently, any criterion which is established by means of parametric variations can only be defined as ultimate when it is the most valid one from geometric considerations.

From the black body definition

$$J_\lambda(T;\ C_1,\ C_2) \;=\; C_1\ \bar{J}_\lambda(T;\ C_2) \;=\; \frac{C_1}{\lambda^5 \left(e^{C_2/\lambda T} - 1\right)}\ , \qquad (1)$$

where λ is the wavelength in cm, T is the temperature in °K, and C_1 and C_2 are constants (or parameters). Any of the quantities T, C_1, and C_2 may be chosen as a parameter. First consider

*Mr. Fussell is now working for the National Bureau of Standards, Washington, D. C.

variations in C_1. It is clear that any change in the magnitude of C_1 shifts the spectral distribution up or down along the ordinate depending on the sign of the chosen change for C_1. Since the parameters C_2 and T are geometrically equivalent (in a reciprocal manner), only one of them need be considered; T was chosen because of its experimental accessibility. Variations in T affect the shape of the black body distribution and, of course, the wavelength at which the maximum spectral intensity occurs. Therefore, any criterion which is to be established for fitting the sun's extraterrestrial spectral distribution to Equation 1 can be determined by means of parametric variations in T and C_1, or a gray body fit.

The most straightforward approach is to find the temperature at which the black body maximum matches the sun's maximum, 0.475 micron. This temperature is readily found to be 6060°K. But this method gives a black body curve which contains a greater integrated energy than the sun's integrated spectral distribution, since by definition the black body distribution gives an upper limit to the emissive ability of any body at a given temperature. Consequently, an upper bound on the required temperature has been established from this matching process. Next, an estimate of the lower bound can be found. To accomplish this, C_1 and T may be varied in an effort to minimize the residual

$$R = \int |J_\lambda(T;\ C_1,\ C_2) - H_\lambda|\, d\lambda \quad , \tag{2}$$

where H_λ and the vertical lines represent the sun's spectral distribution and absolute magnitude, respectively. From Equation 2 the lower bound is found to be approximately 5738°K. It is apparent that Equation 2 alone cannot give the required closeness of fit to the desired accuracy because one criterion cannot be used to minimize a bivariant distribution. The final criterion, the one defined as geometrically ultimate and, consequently, most pertinent from the thermal design viewpoint, is the system:

$$R_{Min} = \mathrm{Min} \int |J_\lambda(T;\ C_1,\ C_2) - H_\lambda|\, d\lambda \tag{3}$$

and

$$\int J_\lambda(T;\ C_1,\ C_2)\, d\lambda = \int H_\lambda\, d\lambda \quad . \tag{4}$$

The addition of Equation 4, to establish the most meaningful criterion, allows a temperature to be found by using matched areas as the basis. The absolute magnitude in Equation 3 is necessary in order to insure that the total deviation between the two curves is minimized. From Equation 1 it is clear that C_1 can be eliminated in Equation 3 by means of Equation 4, which gives a single relation for the determination of T. These equations give a temperature of 5742 ± 0.5 °K and a value of 8.65040×10^{-17} watt-cm^2 for C_1 for the 0.3 to 2.4 micron (μ) region.

It remains to show that Equation 2 should give results identical to Equations 3 and 4, indicating therefore that 5742°K is the desired temperature. If the spectral region of interest is broken into intervals of $J_\lambda(T; C_1) > H_\lambda$ and $J_\lambda(T; C_1) < H_\lambda$, where the intervals of the first kind are designated by $\Delta_{1,1}$ and those of the second kind by $\Delta_{2,1}$, then Equations 2 and 3 may be rewritten:

$$\int_{\Delta_{1,1}} |J_\lambda(T; C_1) - H_\lambda| d\lambda + \int_{\Delta_{2,1}} |J_\lambda(T; C_1) - H_\lambda| d\lambda$$

$$= \int_{\Delta_{1,1}} C_1 \bar{J}_\lambda(T) d\lambda - \int_{\Delta_{1,1}} H_\lambda \, d\lambda - \int_{\Delta_{2,1}} C_1 \bar{J}_\lambda(T) d\lambda + \int_{\Delta_{2,1}} H_\lambda \, d\lambda \quad , \tag{5}$$

where the first subscript of Δ refers to the interval type and the second is an index which enumerates subintervals. When the derivatives of the right-hand side of Equation 5 (ignore Equation 4) are taken with respect to C_1 and T in an effort to obtain simultaneous equations for C_1 and T, the following results are obtained:

$$\int_{\Delta_{1,1}, \Delta_{2,1}} \bar{J}_\lambda(T) d\lambda = 0$$

and

$$C_1 \int_{\Delta_{1,1}, \Delta_{2,1}} \frac{\partial}{\partial T} \bar{J}_\lambda(T) = 0 \quad .$$

Consequently, Equation 4 must be included in order to define C_1 and obtain the desired results. This implies the equivalence of Equation 2 and the system of Equations 3 and 4. By taking the derivative of the right-hand side of Equation 5 with respect to T and equating the result to zero in order to obtain a minimum (make use of Equation 4), Equation 5 may be rewritten:

$$\int_{\Delta_{1,1}} |J_\lambda(T; C_1) - H_\lambda| d\lambda + \int_{\Delta_{2,1}} |J_\lambda(T; C_1) - H_\lambda| d\lambda \le \epsilon \quad , \tag{6}$$

where ϵ is a number characterizing the fit of the black body curve to the sun's distribution and $\Delta_{1,1}$ and $\Delta_{2,1}$ become the sets $\{\Delta_{1,i}\}$ and $\{\Delta_{2,j}\}$, respectively, with $1 \le i \le N$ and $1 \le j \le M$. If the fit is exact, $\epsilon = 0$ and the equality sign holds; however, if the fit is not exact, the left-hand side of Equation 5 gives the best fit, which means that Equations 3 and 4 give the desired temperature.

RESULTS

The results given after each method of calculation described in the previous section are for the wavelength range of 0.3-2.4 μ. Calculations were also made for the intervals 0.4-1.2 and 0.220-7.0 μ;

the former region is of interest for purposes of solar cell illumination, and the latter is a complete cover of Johnson's data.* All results are given in Table 1. The auxiliary range of 0.3-2.0μ is included for comparison purposes.

It is apparent that the best fit (2.97 percent) was obtained for the 0.4-1.2μ region, which indicates that the adjusted black body curve conforms reasonably well to Johnson's data throughout the visible and into the near infrared. Less satisfactory agreement (6.08 percent) was obtained for the region of 0.3-2.0μ because a single black body formula cannot be made to fit the steep rise in emissiveness characteristic of the sun's radiation in the region of 0.34-0.4μ. Agreement is enhanced, however, when the region is extended to 2.4μ because the additional 2.0-2.4μ region, where the fit is good, makes the subvisible region (0.34-0.4μ) of somewhat less weight. The fit for the complete range covered by Johnson's data, 0.220-7.0μ, is least satisfactory. The black body curve (with only two adjustable parameters) cannot be made to fit the steep rise in the 0.220-0.4μ region as well as the tail of the sun's spectral distribution in the infrared.

In order to clarify the fit to the solar extraterrestrial spectrum by the above gray body matching technique, a gray body curve was calculated for the 0.3-2.4μ region. The gray body curve of this region for a temperature of 5742°K is compared with the solar data in Figure 1. According to column 6 of Table 1 the areal mismatch of these curves is 5.94 percent when computed relative to the solar constant. It can be concluded that a gray body curve agrees well with the solar data curve, except for the irregular region of 0.41-0.55μ. What has happened is that virtually exact fits occurred at 0.43 and 0.52μ, and the intermediate irregularities could not be fit by a single gray body curve.

Table 1

Results of Gray Body Fit to Solar Extraterrestrial Spectrum.

Range (microns)	T (°K)	C_1 (watt-cm^2)	$\int H_\lambda d\lambda$ (watt/cm^2)	$\int \lvert J_\lambda - H_\lambda \rvert d\lambda$ (watt/cm^2)	$\frac{\int \lvert J_\lambda - H_\lambda \rvert d\lambda}{\int H_\lambda d\lambda} \times 100$
0.4 - 1.2	6142 ± 0.5	6.76111×10^{-17}	9.86137×10^{-2}	2.92699×10^{-3}	2.97
0.3 - 2.0	5730 ± 0.5	8.72019×10^{-17}	1.29419×10^{-1}	7.86669×10^{-3}	6.08
0.3 - 2.4	5742 ± 0.5	8.65040×10^{-17}	1.32629×10^{-1}	7.88560×10^{-3}	5.94
0.220 - 7.0	5693 ± 0.5	8.80334×10^{-17}	1.39424×10^{-1}	1.03376×10^{-2}	7.41

DISCUSSION

If the black body approximation to the solar spectrum is to be useful from the thermal design viewpoint, the average absorptance of a spacecraft coating material, as calculated from the black body spectrum, should be fairly close in value to the average solar absorptance of the material. Mathematically expressed, it is desirable that $\bar{a}_{BB} \approx \bar{a}_s$, where

*Johnson, F. S., "The Solar Constant," *J. Meteorology* 11(6): 431-439, December 1954.

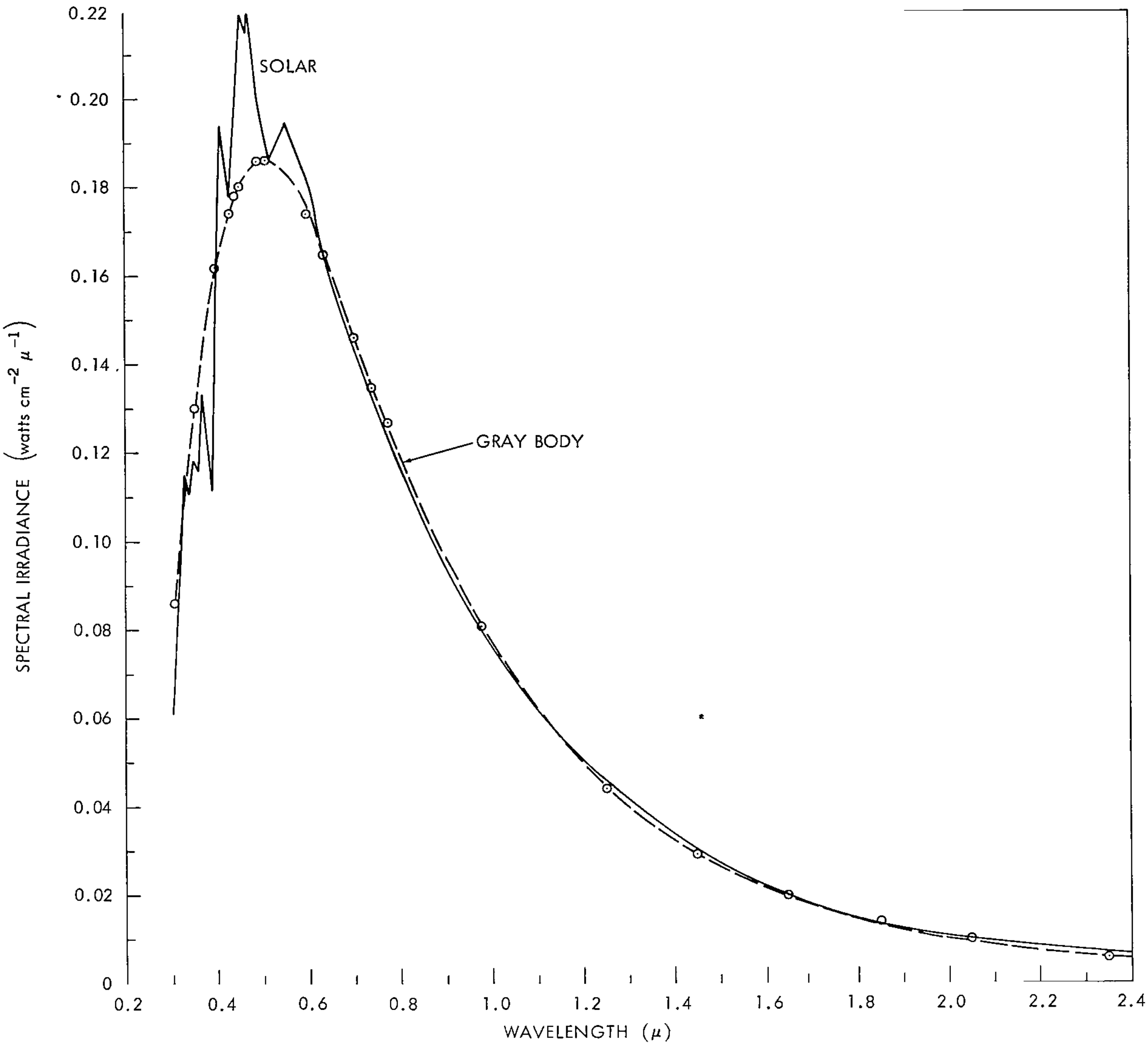

Figure 1—Comparison of a 5742°K gray body curve with the solar extraterrestrial spectrum.

$$\bar{a}_{BB} \equiv \frac{\int_0^\infty a_\lambda J_\lambda(T)d\lambda}{\int_0^\infty J_\lambda(T)d\lambda}, \tag{7}$$

and

$$\bar{a}_s \equiv \frac{\int_0^\infty a_\lambda H_\lambda d\lambda}{\int_0^\infty H_\lambda d\lambda}, \tag{8}$$

with

$\bar{a}_{BB}$ = the average absorptance for a black body,

$\bar{a}_s$ = the average solar absorptance,

H_λ = the solar spectral intensity per unit wavelength bandwidth, as given by Johnson,

$J_\lambda(T)$ = the Planck black body spectral intensity for absolute temperture T,

a_λ = the absorptance of the material at wavelength λ.

For simplicity, it is desirable to normalize H_λ and J_λ to H'_λ and J'_λ :

$$H'_\lambda = \frac{H_\lambda}{\int_0^\infty H_\lambda \, d\lambda} ,$$

$$J'_\lambda = \frac{J_\lambda}{\int_0^\infty J_\lambda \, d\lambda} .$$

Equations 7 and 8 now become

$$\bar{a}_{BB} = \int_0^\infty a_\lambda J'_\lambda(T) d\lambda , \tag{9}$$

and

$$\bar{a}_s = \int_0^\infty a_\lambda H'_\lambda d\lambda . \tag{10}$$

Arbitrarily, a 10 percent proportional difference in $\bar{a}_{BB}$ and $\bar{a}_s$ will be chosen as the criterion of usefulness; that is, if

$$\frac{|\bar{a}_{BB} - \bar{a}_s|}{\bar{a}_s} \equiv E_p \leq 0.1 , \tag{11}$$

the black body approximation will be considered useful. Equation 11 is actually

$$\frac{\left| \int_0^\infty \left[J'_\lambda(T) d\lambda - H'_\lambda \right] a_\lambda d\lambda \right|}{\int_0^\infty a_\lambda H'_\lambda \, d\lambda} \leq 0.1 . \tag{12}$$

It can be seen that the results given in column 5, Table 1 for the quantity

$$\int \left| J'_\lambda(T) - H'_\lambda \right| d\lambda \equiv \Delta_p \tag{13}$$

do not indicate the magnitude of E_p (Equation 11). Consider now the following two cases:

1. If a_λ is approximately constant, then the proportional error E_p becomes

$$E_p = \frac{a_\lambda \left| \int_0^\infty \left[J'_\lambda(T) - H'_\lambda \right] d\lambda \right|}{a_\lambda \int_0^\infty H'_\lambda d\lambda} \; ; \tag{14}$$

and, since

$$\int_0^\infty J'_\lambda(T) d\lambda = \int_0^\infty H'_\lambda d\lambda = 1 \quad , \tag{15}$$

it is clear that

$$\int_0^\infty \left[J'_\lambda(T) - H'_\lambda \right] d\lambda = 0 \tag{16}$$

and $E_p = 0$.

2. If a_λ can be represented by a power series in λ, that is,

$$a_\lambda = \left(a_\lambda\right)_0 + \left(\lambda - \lambda_0\right)\left(\frac{da_\lambda}{d\lambda}\right)_0 + \frac{\left(\lambda - \lambda_0\right)^2}{2!}\left(\frac{d^2 a_\lambda}{d\lambda^2}\right)_0 + \cdots \quad , \tag{17}$$

then

$$E_p = \frac{\left| \int_0^\infty \left[J'_\lambda(T) - H_\lambda \right] \lambda \left(\frac{da_\lambda}{d\lambda}\right)_{0,\,i} d\lambda \right|}{\int_0^\infty H'_\lambda a_\lambda d\lambda} \quad , \tag{18}$$

in which the error is now weighted by the slope.

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—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

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